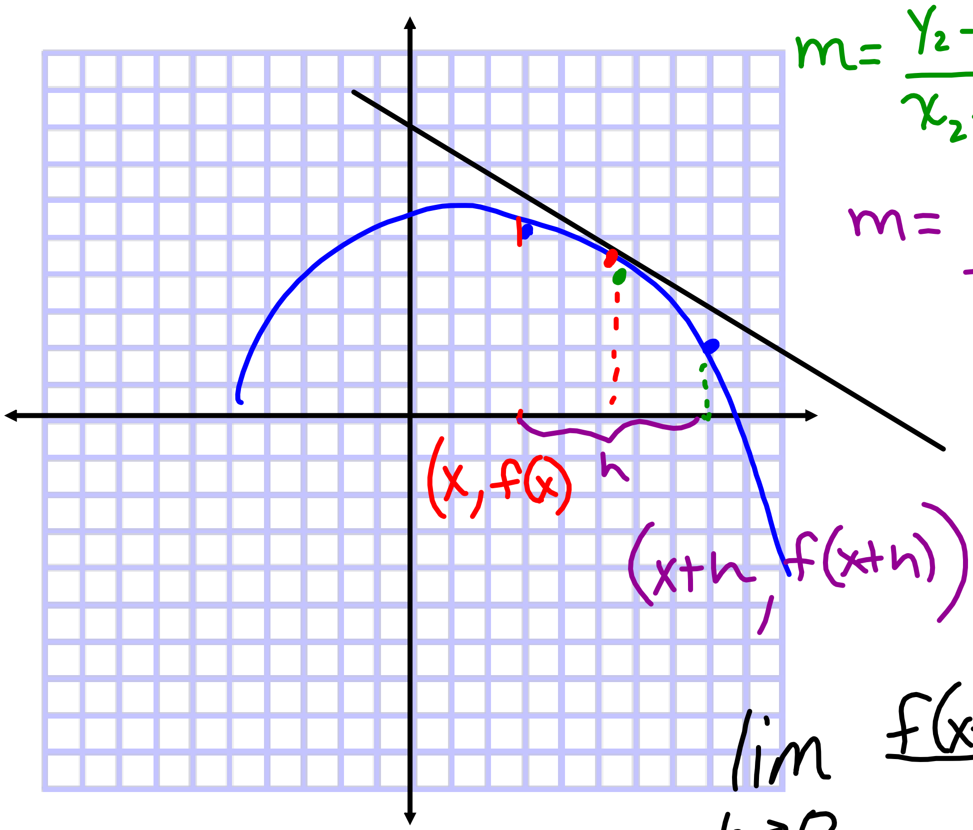
The Tangent Line



$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{\overset{y_2}{f(x+h)} - \overset{y_1}{f(x)}}{\underset{x_2}{\cancel{x}+h} - \underset{x_1}{\cancel{x}}}$$

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Derivative:

* not the equation of the tangent line

A formula for the slope of the tangent line to the graph at a certain point.

$f'(x)$, $\frac{dy}{dx}$, $\frac{df(x)}{dx}$, $D_x[y]$

$f''(x)$

Equation of a line:

slope

$y = mx + b$

yint

Point-slope:

$(y - y_1) = m(x - x_1)$

$\downarrow$

$y = mx + b$

1) use a derivative to find slope

2) find the point

3) equation of line

Find the equation of the tangent line at $x=\frac{3}{4}$.

$$f(x)=\frac{1}{2x-1}$$

1)

$$\frac{\frac{1}{2(x+h)-1}-\frac{1}{2x-1}}{h} \rightarrow \frac{\frac{1}{2x+2h-1}-\frac{1}{2x-1}}{h}$$

$$\lim_{h\to 0}\frac{\cancel{2x}\cancel{-1}\cancel{-2x}-2h\cancel{+1}}{(2x\cancel{+2h}-1)(2x-1)}\cdot\frac{1}{\cancel{h}} \rightarrow \frac{-2}{(2x-1)^2}$$

$$m=\frac{-2}{(\cancel{2}\cdot\frac{3}{\cancel{4}_2}-1)^2} \rightarrow \frac{-2}{\frac{1}{4}} = -8$$

slope @ $x=\frac{3}{4}$

2) Point → $\left(\frac{3}{4}, 2\right)$ $\quad \frac{1}{\cancel{2}\left(\frac{3}{\cancel{4}_2}\right)-1}$

3)

$$y-2=-8\left(x-\tfrac{3}{4}\right)$$

$$y-2=-8x+6$$

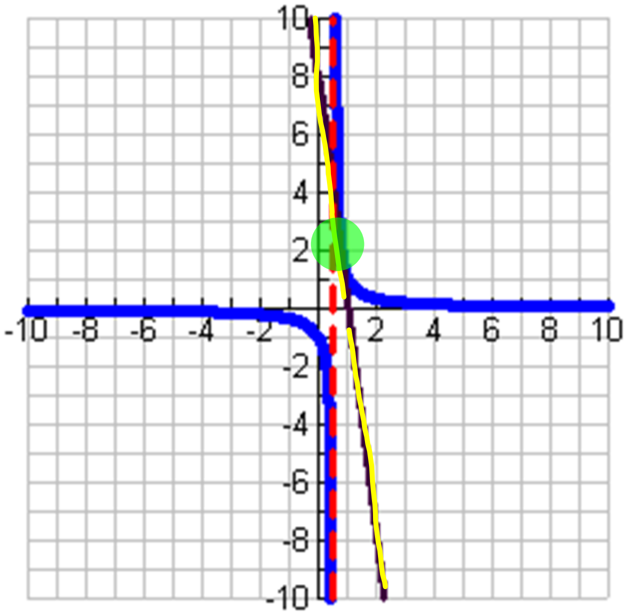
$$y=-8x+8$$

$$f(x)=\frac{1}{2x-1}$$

$$f'(x)=\frac{-2}{(2x-1)^2}$$

$$f'\left(\frac{3}{4}\right)=-8$$

$$f\left(\frac{3}{4}\right)=2$$



Find the equation of the tangent line at x=2.

$$f(x) = -2\sqrt{x-1}$$

1) Slope → $\dfrac{-2\sqrt{x+h-1} + 2\sqrt{x-1}}{h}$

$\dfrac{-2\sqrt{1+h} + 2}{h} \cdot \dfrac{(-2\sqrt{1+h} - 2)}{(-2\sqrt{1+h} - 2)}$

$\lim_{h \to 0} \dfrac{4(1+h) - 4}{h(-2\sqrt{1+h} - 2)} \rightarrow \dfrac{\cancel{4} + 4\cancel{h} \cancel{-4}}{\cancel{h}(-2\sqrt{1+\cancel{h}} - 2)}$

$-2 - 2$

$m = \dfrac{4}{-4} = -1$

2) Point: $(2, -2)$ $\quad -2\sqrt{2-1}$

3) $y + 2 = -1(x - 2)$

$y = -x$

Find the equation of the tangent line at x=2

$$f(x) = 3x^2 - 2x + 1$$

1) $\dfrac{3(x+h)^2 - 2(x+h) + 1 - 3x^2 + 2x - 1}{h}$

$\dfrac{\cancel{3x^2} + 6xh + 3h^2 \cancel{- 2x} - 2h \cancel{+1} \cancel{- 3x^2} \cancel{+ 2x} \cancel{- 1}}{h}$

$\lim\limits_{h \to 0} \dfrac{\cancel{h}(6x + 3\cancel{h} - 2)}{\cancel{h}}$

$6x - 2$

$m = 12$

2) Point → (2, 9) $3(2)^2 - 2(2) + 1$
$12 - 4 + 1$

3) $y - 9 = 12(x - 2)$

$y = 12x - 15$

Find the derivative. Use the derivative to determine any points on the graph f(x) at which the tangent line is horizontal.

$f(x) = x^2 - 4x + 3$

$$\frac{(x+h)^2 - 4(x+h) + 3 - x^2 + 4x - 3}{h}$$

$$\frac{\cancel{x^2} + 2xh + h^2 \cancel{-4x} - 4h \cancel{+3} \cancel{-x^2} \cancel{+4x} \cancel{-3}}{h}$$

$$\lim_{h \to 0} \frac{\cancel{h}(2x + h - 4)}{\cancel{h}} = 2x - 4$$

Slope is zero for a horizontal line

$2x - 4 = 0$

$x = 2$

Point $(2, -1)$

$2^2 - 4(2) + 3$

$4 - 8 + 3$